Evolution of Leaky Modes on Printed-Circuit Lines

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***Abstract*—The frequency evolution of dominant (quasi-TEM) and higher order modes on an open printed-circuit structure such as a microstrip is examined. Three different mode types are considered, including bound modes (BMs), leaky modes that leak into the surface wave of the background structure, and leaky modes that also leak into space. One of the fundamental goals is to establish the conditions under which one type of mode can transition into another type as the frequency changes. One important conclusion is that the dominant BM can never transition into a leaky mode for a microstrip structure with an isotropic substrate, but such a transition is possible for an anisotropic substrate, observed originally by Tsuji *et al.*and Shigesawa *et al.* However, higher order BMs can directly transition into leaky modes, as shown by Oliner and Michalski and Zheng. On other structures such as coplanar strips, where the bound dominant mode exhibits odd symmetry, a transition from a bound dominant mode to a leaky mode is possible, as shown by Shigesawa *et al.* and Tsjui *et al.*. In addition to examining the mathematical transitions that are possible, the physical continuation of modes is also investigated, by examining the frequency evolution of the currents excited by a practical source. It is concluded that there may be physical continuity between modes, even if there is no mathematical continuity.**

***Index Terms*—Leaky modes, microstrip lines, spectral-domain techniques.**

I. Introduction

THE presence of leaky modes in the electromagnetic spectrum of printed-circuit lines has motivated a lot of work focused on examining their properties [1]–[28]. From early on, leaky modes were recognized as being responsible for undesirable radiation and crosstalk in guiding structures. In general, leaky modes on printed-circuit lines can be divided in two types: surface-wave leaky modes (SFWLMs) and space-wave leaky modes (SPWLMs). The SFWLMs are those modes that leak power in the form of surface waves on the background waveguiding structure (usually the TM_0 surface wave) as they propagate. SPWLMs leak power into free space in addition to that into the surface waves. As is explained in [11], [15], [17], and [26], the above two different mechanisms of power leakage are reflected in the spectral domain analysis (SDA) of the leaky modes by the use of different integration paths in the transverse wavenumber plane that detour around different singularities of the spectral dyadic Green's function (SDGF). Specifically, the integration contour of an SFWLM detours only around the poles of the SDGF that are associated with the wavenumbers of the background waveguide modes into which leakage takes places. For an SPWLM, the integration path detours also around the branch points of the SDGF associated with the free-space wavenumber. A bound mode (BM) is computed using an integration path that does not detour around any singularities of the SDGF, namely, the integration path runs along the complete real axis.

Depending on the structure, it has been observed in the past that leaky modes can coexist with the dominant and higher order BMs of the structure [14], [17] or a leaky mode can be the dominant quasi-TEM mode of the structure [19], [21]. Due to the simultaneous presence of BMs and different types of leaky modes, many researchers have studied the modal behavior for various structures and the relations and connections between the different types of modes as frequency (or some other structural parameter) changes. Thus, it has been found that the dominant BM of a microstrip line never becomes leaky [6], except when the substrate has anisotropic characteristics [12]. However, for higher order modes on a microstrip, transitions from a BM to an SFWLM do occur, and behave similarly to the situation when a BM transitions to a leaky mode in a simple dielectric-slab waveguide [30], involving the so-called spectral gap region [14], [16]. For other structures, such as coplanar strips and coplanar waveguide, the BM may transition into a leaky mode in a similar way [14], [7]. In all cases, a direct transition from a bound to an SPWLM has never been observed. Another transition that has never been reported is an SFWLM evolving into an SPWLM.

Almost all the above investigations were carried out using numerical solutions to the dispersion equations of the printed-circuit line, which provides accurate solutions, but does not provide a fundamental understanding of why certain transition are possible and others are not. The aim of this paper is to study the possible transitions of bound and leaky modes on printed-circuit lines with the purpose of explaining the evolution and possible transitions of these modes. It will be clarified what conditions allow for possible transitions between one type of mode and another (i.e., necessary conditions for these transitions). The methodology used in this investigation is based on the insight provided by the Riemann surface defined in the complex longitudinal wavenumber plane that is associated with the solution of a printed-circuit line with a practical source excitation. This plane has proved to be a very useful tool for exploring and understanding what transitions are possible for the modes on an infinite line, as well as allowing for a convenient numerical solution to the currents and fields due to a practical source excitation.

Section II presents a brief overview of the SDA when applied to both a two-dimensional (2-D) analysis of an infinite printed-circuit line and a three-dimensional (3-D) analysis of

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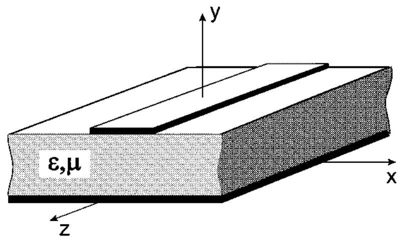


Fig. 1. Microstrip on an isotropic and lossless substrate excited by a delta-gap voltage source.

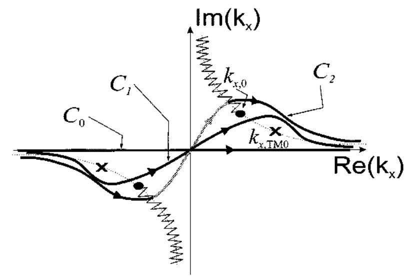


Fig. 2. Possible integration paths C_x. The integration paths all lie on the proper sheet of the k_0 branch points, except for the grey part of the C_2 path, which lies on the improper sheet.

the line when excited by a source. This theory is used in Section III to explain the conditions under which various types of transitions from one mode to another may take place. Numerical results for dispersion behavior on a microstrip line are presented to illustrate the conclusions. Section IV will present numerical results to illustrate another important point, namely, that the lack of mathematical continuation between modes does not necessarily means the absence of a possible physical continuity. For example, the most prominent leaky mode that is physically present on a microstrip line due to a practical source excitation may change from an SFWLM to an SPWLM as the frequency changes, even though there is no mathematical connection between these two modes.

II. Analysis

The SDA has proven to be one of the most efficient and fruitful techniques to study the dispersion characteristics of printed-circuit lines [4], [11], [13], [15], [16], [17], [19]. As is explained in the literature, the Galerkin method in conjunction with the Parseval's theorem can be used to pose the dispersion relation of an infinite printed-circuit line (e.g., the microstrip line shown in Fig. 1) as the zeros of the following equation:

$$F(k_z) = \int_{C_x} \widetilde{G}_{zz}(k_x; k_z)\widetilde{T}^2(k_x)\, dk_x = 0 \qquad (1)$$

where $\widetilde{T}(k_x)$ is the Fourier transform of the basis function $T(x)$ used to expand the longitudinal current density on the strip conductor as

$$J_{sz}(x, z) = T(x)e^{-jk_z z}. \qquad (2)$$

The term $\widetilde{G}_{zz}(k_x; k_z, \omega)$ is the zz component of the SDGF, and C_x is an appropriate integration path in the complex k_x-plane to allow for an inverse Fourier transform of nonuniformly convergent functions [31], [32]. (In the derivation of (1), only one basis function was used for simplicity; the use of more basis functions does not affect the general conclusions discussed below). The SDGF has the following singularities on the k_x-plane.

1) Branch points at $k_{x,0} = \pm\sqrt{k_z^2 - k_0^2}$, with k_0 being the free-space wavenumber.These branch points define a two-sheeted Riemann surface in the k_x-plane. Using the Sommerfeld choice for defining the corresponding branch cuts, $\mathrm{Im}(k_{y0}) = \mathrm{Im}\sqrt{k_0^2 - k_x^2 - k_z^2} = 0$, the two sheets correspond to the vertical wavenumber k_{y0} being proper (imaginary part negative) and improper (imaginary part positive).
2) A finite set of poles on the proper sheet at $k_{x,\mathrm{sw}} = \pm\sqrt{k_z^2 - k_{\mathrm{sw}}^2}$, where k_{sw}^2 is the square of the wavenumber of the above cutoff proper modes (surface waves) of the background waveguide.
3) An infinite set of poles on the improper sheet at $k_{x,\mathrm{imp}} = \pm\sqrt{k_z^2 - k_{\mathrm{imp}}^2}$, where k_{imp}^2 is the square of the wavenumber of the real and complex improper modes of the background waveguide.

For a fixed frequency, the function $F(k_z)$ is not uniquely defined because of the many possible different C_x integration paths that can be used to carry out the integral in (1). The different C_x paths come from the different singularities of the SDGF that can be detoured around. It is well known that a BM solution comes from using the real-axis path of integration in (1). For complex leaky-mode solutions, an integration path detouring around only the proper (surface-wave) poles of the SDGF is associated with an SFWLM solution. If the path also detours around the branch points, passing through the branch cuts and, therefore, lying partly on the lower Riemann sheet, the path will be associated with an SPWLM solution. The three different types of paths are shown in Fig. 2. For convenience, the three different types of path will be denoted as C_0 for the real-axis path (BM solution), C_1 for the path that detours around only the SDGF poles (SFWLM solution), and C_2 for the path that also passes around the branch points (SPWLM solution).

The nonuniqueness in the choice of the integration paths in the k_x-plane causes the function $F(k_z)$ to be multivalued. In particular, branch points appear in the k_z-plane. A location on a particular sheet in the k_z-plane corresponds to a particular choice of integration path in the k_x-plane in (1). An understanding of the geometry of the complex k_z-plane is central to the later discussion on the types of modal transitions that are allowed. The k_z-plane also provides insight into when leaky modes are physically significant, in the sense that the mode appears to a significant degree in the current spectrum excited by a practical source.

The complex k_z-plane for printed-circuit structures such as microstrip is examined in detail in [28]. It is concluded there that the following types of branch points exist in the k_z-plane:

1) logarithmic (infinite-sheeted) branch points at $k_z = \pm k_0$;
2) square-root-type (two-sheeted) branch points at $k_z = \pm k_{\mathrm{sw}}$ located on the even sheets with respect to the k_0 branch point;
3) square-root-type branch points at $k_z = \pm k_{\mathrm{imp}}$ located on the odd sheets with respect to the k_0 branch point.

The third type of branch point will be ignored here since it is not relevant for this discussion. Assuming one surface-wave mode

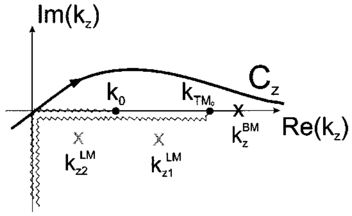


Fig. 3. Singularities appearing in the k_z complex plane. The branch cuts associated with the branch points at k_0 and k_{TM_0} are shown, along with a BM pole and two leaky-mode poles. The path of integration used in the solution of the current excitation by a delta-gap source is also shown.

(the TM_0 mode) above cutoff, and using Sommerfeld hyperbolic branch cuts, the branch cuts in the k_z-plane are as shown in Fig. 3.

The use of different integration paths in the k_x-plane provides for the possibility of many possible mathematical leaky-mode solutions, corresponding to wavenumbers k_z that appear on different sheets of the branch points in the k_z-plane. However, only a few of them will have physical significance. This question was extensively treated in [27] and [28], where it was shown that a pure 2-D analysis cannot satisfactorily answer this point. In those papers, a leaky mode was considered physically significant provided that its associated current appears in the spectrum of the total current $I(z)$ excited by a realistic source on the line. Assuming that a delta-gap voltage source is feeding the line (with an impressed field $E_z^{\mathrm{gap}}(z)$), the solution to the corresponding electric-field integral equation (EFIE) in the spectral domain makes it possible to express the total current as the following inverse k_z Fourier transform [28]:

$$I(z) = \int_{C_z} \frac{E_z^{\mathrm{gap}}(k_z)}{F(k_z)}\, e^{-jk_z z}\, dk_z. \tag{3}$$

The path of integration in the k_z-plane is shown in Fig. 3. The path stays on the zero sheet of the k_0 branch point and the top sheet of the k_{TM_0} branch point. By definition, this is equivalent to a real-axis path in the k_x-plane. The denominator of the integrand in the above equation is precisely the function $F(k_z)$ defined in (1), thus, the branch points and poles of the integrand come from the branch points and roots of $F(k_z)$. This causes the spectrum of the total current produced by the source to be clearly dependent of the singularities in the complex k_z-plane. In particular, pole singularities of the integrand, corresponding to the roots of the function $F(k_z)$, determine the discrete spectrum of guided modes launched on the line by the delta-gap source. The residues of the integrand at the pole singularities determine the launching amplitudes of the guided modes, both bound and leaky. A BM pole with a real propagation wavenumber (on the real k_z-axis), and two typical complex leaky-mode poles are shown in Fig. 3.

Not all of the pole singularities are expected to be physically significant, in that the corresponding guided mode appears in the spectral representation of the current produced by the source. It was shown in [28] that a path consistency condition (PCC) must be met in order for a leaky-wave solution to be physically significant. The PCC provides a systematic criterion to determine if a mathematical solution to the dispersion equation of the infinite structure is expected to be physically excited by a practical source. To summarize, the PCC states that, in order for a leaky mode to be physical, the value of the phase constant β obtained by using a certain path of integration C_x must be *consistent* with the path. For example, the leaky-mode solution corresponding to the pole k_{z1}^{LM} in Fig. 3 has a phase constant β that is between k_0 and k_{TM_0}. This solution will, therefore, only be physical if it arises from an integration path in the k_x-plane of the type C_1, meaning that it is an SFWLM type of solution. This is also equivalent to saying that the pole k_{z1}^{LM} lies on the zero sheet of the k_0 branch point and the lower sheet of the k_{TM_0} branch point. Similarly, the leaky mode associated with k_{z2}^{LM} has a phase constant β less than k_0 and, hence, it will be physical provided that its corresponding integration path in the k_x-plane is C_2, meaning that it is an SPWLM type of solution. Equivalently, the pole is located on the lower (-1) sheet of the k_0 branch point.

III. Continuity of Solutions From Different Paths

In the study of the dispersion relations for printed-circuit lines, it is usually found that different mathematical leaky-wave solutions are physically meaningful only within certain frequency ranges. This raises the question of the possible mathematical and physical continuity between the different leaky-wave solutions as frequency (or some other parameter) changes. The issue of mathematical continuity of different modes can be conveniently explored by tracking the loci of the corresponding propagation constants along the different sheets of the k_z complex plane. In Section II, it was shown that the different types of solutions to (1) are located on different sheets. Thus, one solution can be the continuation of other type of solution only if the solution crosses the corresponding branch cut. Based on this consideration, some possible transitions will be studied next. Specifically, some of the questions that will be examined are under what conditions a bound dominant mode or higher order BM can ever evolve into an SFWLM type of leaky mode, and if an SFWLM type of leaky mode can ever evolve into an SPWLM type of leaky mode.

A. Transition From BM Solutions to Surface-Wave Leaky Solutions

Since the solutions from path C_0 do not leak in any form, these solutions will be evidently associated with the dominant and higher order BM solutions. In the past, different behaviors have been observed for BM solutions. The dominant mode of an isotropic microstrip line has never been observed to evolve to an SFWLM [6] (i.e., it never becomes leaky), although such an evolution has been observed for a microstrip line on an anisotropic substrate [8]. Dominant BMs on other structures with isotropic substrates, such as coplanar strips and coplanar waveguides, have been found to evolve into an SFWLM type of solution [7], [11], [14], [18]. In a lossless structure the BMs have real propagation constants greater than the wavenumber of the TM_0 surface-wave mode of the background waveguide. Hence, if a BM solution transitions into a leaky-mode solution, it must evolve into the leaky solution by first passing through the rightmost branch point located (the k_{TM_0} branch point) in Fig. 3 to reach one of the bottom sheets. The criterion for which this is possible will be discussed momentarily.

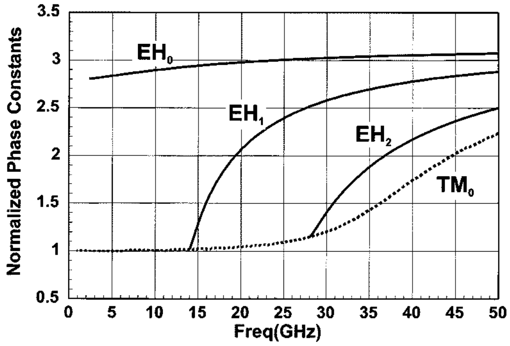


Fig. 4. Plots of the normalized phase constants (β/k_0) for the EH_0, EH_1, and EH_2 modes of a microstrip line on an isotropic substrate with $h = 0.635$ mm, $w = 3$ mm, and $\epsilon_r = 9.8$. The TM_0 surface wave of the grounded dielectric slab is also shown.

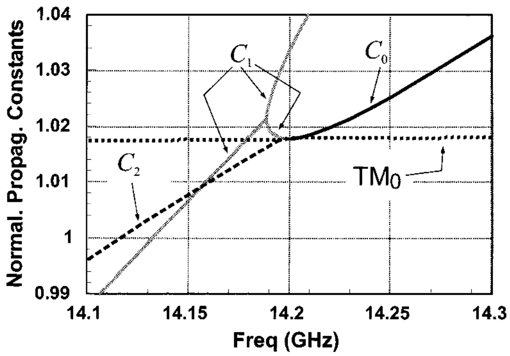


Fig. 5. Detailed plot of the normalized phase constants showing the transition for the EH_1 mode in Fig. 4. The C_0 (BM) solution turns into a real improper C_1 (surface-wave leaky) solution prior to merging with another real improper solution to give rise to a complex C_1 solution. A C_2 solution, which is not the mathematical continuation of the bound EH_1 mode, is also shown in this figure.

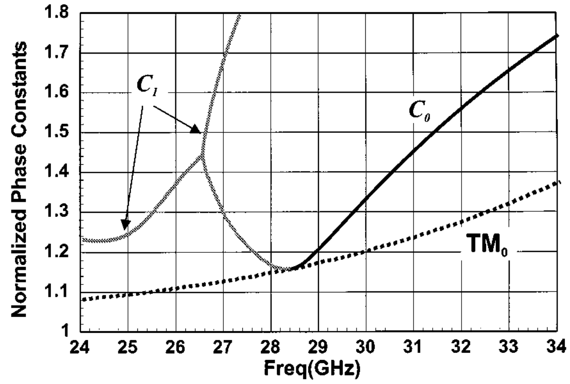


Fig. 6. Detailed plot of the transition for the EH_2 mode shown in Fig. 4. The C_0 (BM) solution turns into a real improper C_1 (surface-wave leaky) solution prior to merging with an other real improper solution to give rise to a complex C_1 solution.

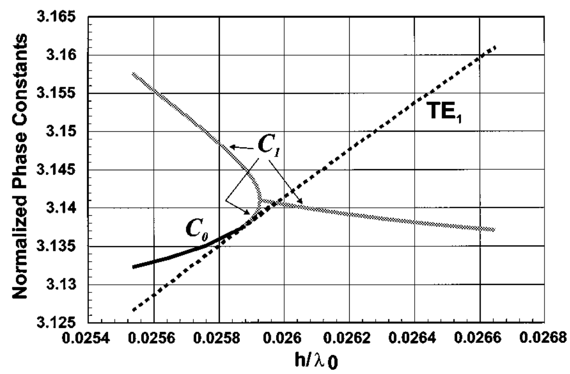


Fig. 7. Detailed plot of the of the normalized phase constants showing the transition for the EH_0 BM studied in [8, Fig. 2(a)]. The C_0 (BM) solution turns into a real improper C_1 (surface-wave leaky) solution that meets another real improper solution to give rise to a complex C_1 solution.

The evolution of the bound C_0 solutions for the dominant and first two higher order modes, i.e., the EH_0, EH_1 and EH_2 modes, of a microstrip line with an isotropic substrate is shown in Fig. 4. It is observed in this figure that the dominant EH_0 solution never crosses the TM_0 dispersion curve to become an SFWLM, whereas the EH_1 and EH_2 bound higher order modes do cross the TM_0 dispersion curve (at 13.5 and 28 GHz, respectively), and evolve into SFWLM solutions at lower frequencies. Specifically, the bound solutions first make a transition to a real improper SFWLM solution, arising from path C_1, which meets another real improper SFWLM solution to then give rise to a complex (leaky) SFWLM solution. This behavior is shown in Fig. 5 for the EH_1 mode (which has a longitudinal current distribution $T(x)$ that is an odd function). The phase constant of the EH_1 leaky SFWLM solution goes below that of the TM_0 surface wave at about 14.18 GHz to become a physically significant leaky mode. This figure also shows that it is possible to find a leaky EH_1 solution by using path C_2. Such a solution leaks into both space and the TM_0 surface wave, and becomes physically significant below about 14.115 GHz, where the curve crosses the unity line ($\beta = k_0$). This C_2 solution is not a mathematical continuation of the C_0 solution (this has been checked carefully), although the C_2 curve approaches closely to the C_0 curve at about 14.2 GHz. The C_2 solution is plotted up to a frequency of about 14.2 GHz, at which point numerical difficulties were encountered due to the close proximity of the C_2 solution to the wavenumber of the TM_0 surface-wave mode (this causes the TM_0 pole in the k_x-plane to approach the origin and, hence, the path of integration).

Similarly, Fig. 6 shows how the C_0 solution for the EH_2 mode [which has an even current distribution $T(x)$] also evolves to a real improper C_1 solution that merges to other similar solution to produce a complex SFWLM solution. Thus, it has been found that both odd and even higher order BMs on a microstrip line with an isotropic substrate make a transition to SFWLM solutions following the same spectral-gap pattern previously reported for other structures [14], [30].

Unlike what is found for the dominant mode of an isotropic microstrip, Tsuji, Shigesawa, and Oliner were the first to report an interesting evolution of the dominant bound C_0 solution into a mode that leaks into only the TE_1 surface wave for a microstrip with an anisotropic substrate [8], [12]. This transition appeared in [8, Fig. 2(a)], which shows how the dominant microstrip mode dispersion curve touches the TM_1 surface-wave curve (which previously had crossed the TM_0 surface-wave mode at a certain frequency) and turns into an SFWLM solution. Reproduced here, Fig. 7 shows how the transition of the BM to the SFWLM solution is made through a spectral gap that is similar to that observed for the higher order modes on an isotropic substrate in Figs. 5 and 6, except that the frequency trend is reversed; leakage occurs at a higher frequency in Fig. 7.

It is important to point out that the transition from the bound to the SFWLM solution for the anisotropic substrate is possible because the TE_1 dispersion curve rises above the TM_0 one. It has never been observed that bound dominant modes on microstrip evolve into SFWLM solutions after crossing the TM_0 dispersion curve. The following proposition rules out this possibility.

Proposition : For printed-circuit lines on an isotropic substrate, the C_0 to C_1 transition ($\mathrm{BM} \rightarrow \mathrm{SFWLM}$) can only occur when the total current on the strip approaches zero as $k_z \rightarrow k_{\mathrm{TM}_0}$.

Note that, unlike a higher order mode, the quasi-TEM dominant BM on microstrip never has a zero total strip current. Hence, the above proposition indicates that the bound dominant mode on microstrip with an isotropic substrate can never transition into an SFWLM, but must remain independent from the leaky modes. Higher order modes on microstrip may transition into SFWLMs (as has already been demonstrated) because they have zero total current. (Odd higher order modes always have zero total current. Even higher order modes may have total zero current, as will be demonstrated later.) Furthermore, the bound dominant mode on structures such as coplanar strips and coplanar waveguides may transition into an SFWLM since such structures support an odd dominant mode with zero total strip current (the sum of the currents on all the conductors is zero).

In order to establish the above proposition, it is first noted that a transition from a BM to an SFWLM requires that $k_z = k_{\mathrm{TM}_0}$ for some frequency, namely, the dispersion curve of the printed-circuit line must touch the dispersion curve of the TM_0 mode. It is assumed that the surface current on the strip conductor $J_{sz}(x,z)$ is represented as shown in (2). The longitudinal electric field at $z=0$ produced by this current density on an infinite line can be expressed as

$$E_z(x) = \frac{1}{2\pi}\int_{C_x} \tilde{E}_z(k_x) e^{-jk_x x} dk_x = \frac{1}{2\pi}\int_{C_x} \tilde{G}_{zz}(k_x;k_z)\tilde{T}(k_x) e^{-jk_x x} dk_x. \quad (4)$$

Assuming that $k_z > k_{\mathrm{TM}_0}$ for a BM, the C_0 integration path is deformed to one running along the branch cut of the SDGF in the lower half of the k_x-plane (for $x>0$) (see Fig. 8) to give the following expression for the electric field

$$E_z(x) = E_z^{\mathrm{TM}_0} + \frac{1}{2\pi}\int_{C_{BC}} \tilde{E}_z(k_x) e^{-jk_x x} dk_x \quad (5)$$

where

$$E_z^{\mathrm{TM}_0} = -j\mathrm{Res}\left\{\tilde{G}_{zz}(k_{x,\mathrm{TM}_0};k_z)\tilde{T}(k_{x,\mathrm{TM}_0})\right\} \times \exp\left(-jk_{x,\mathrm{TM}_0}x\right) \quad (6)$$

accounts for the electric field associated with leakage into the TM_0 surface wave of the background structure, and k_{x,TM_0} is the location in the k_x-plane of the pole of the SDGF associated with the TM_0 surface wave.

As the propagation wavenumber of the BM on the structure approaches the dispersion curve of the TM_0 surface wave, the wavenumber k_z approaches the TM_0 branch point in the k_z-plane, $k_z \rightarrow k_{\mathrm{TM}_0}$). In this limit, $k_{x,\mathrm{TM}_0} \rightarrow 0$. Now taking into account (22), given in the Appendix, the residue of $\tilde{G}_{zz}$ in (6) behaves in this limit as

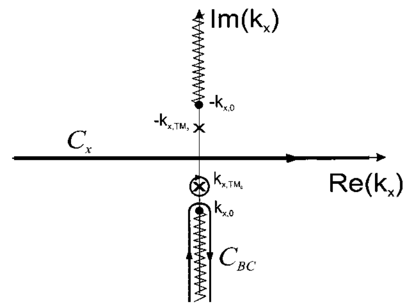


Fig. 8. Deformation of the original C_0 (BM) integration path in the k_x-plane to one residue path around the k_{x,TM_0} pole plus a path detouring around the lower k_{x0} branch cut. In this figure, it has been assumed that $k_z > k_{\mathrm{TM}_0}$ (the pole is then on the imaginary axis).

$$\mathrm{Res}\left\{\tilde{G}_{zz}(k_{x,\mathrm{TM}_0};k_z)\tilde{T}(k_{x,\mathrm{TM}_0})\right\} \propto \frac{\tilde{T}(k_{x,\mathrm{TM}_0})}{k_{x,\mathrm{TM}_0}} \quad (7)$$

and, therefore,

$$E_z^{\mathrm{TM}_0} \propto \frac{\tilde{T}(k_{x,\mathrm{TM}_0})}{k_{x,\mathrm{TM}_0}}. \quad (8)$$

Thus, the electric field associated with the leakage into the TM_0 surface-wave mode tends to infinity unless $\tilde{T}(0)$ is zero. The total current on the strip $I(z)$ may be written as

$$I(z) = \int_{-w/2}^{w/2} J_{sz}(x,z)dx = \left[\int_{-\infty}^{\infty} T(x)dx\right] e^{-jk_z z} = \tilde{T}(0)e^{-jk_z z}. \quad (9)$$

Therefore, the condition $\tilde{T}(0)=0$ implies a zero total current on the strip conductor. Hence, a finite field associated with a propagation wavenumber $k_z = k_{\mathrm{TM}_0}$ is only possible if the total current on the strip is zero. Consequently, a C_0 solution can evolve into a C_1 solution only if the total current is zero at the frequency for which $k_z = k_{\mathrm{TM}_0}$.

The condition of zero total current is always satisfied for an odd mode, such as the EH_1 higher order mode of Fig. 5 for the single microstrip line or the odd dominant mode on coupled microstrip lines (coplanar strips). For higher order *even* BMs, such as the EH_2 mode, the only possibility of finding a transition to a surface-leaky solution is when the total current vanishes as $k_z \rightarrow k_{\mathrm{TM}_0}$. It will be verified next that the transition of the EH_2 mode shown in Fig. 6 is, in fact, possible because $I_{\mathrm{EH}_2}(z) = 0$ as $k_z^{\mathrm{EH}_2} \rightarrow k_{\mathrm{TM}_0}$. After expanding the transverse profile of the longitudinal current, $J_{sz}(x,z)$, in terms of Chebyshev polynomials weighted by a square-root edge singularity term, Fig. 9 shows the solution for the normalized total current computed by means of a 2-D SDA for the EH_2 mode on the microstrip analyzed in Fig. 6. It can be seen that the total current tends to zero as the propagation constant of this mode approaches the wavenumber of the TM_0 surface wave of the background waveguide. This fact is then consistent with the transition from the C_0 solution to the real improper C_1 solution shown in Fig. 6.

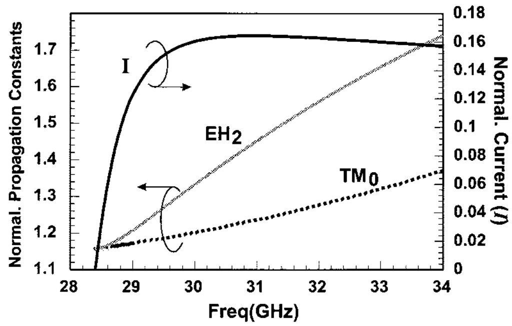


Fig. 9. Plot of the normalized current on the strip conductor versus frequency for the EH_2 mode analyzed in Fig. 6.

There still remains the question of how it is possible to have a transition from the EH_0 BM solution to the SFWLM solution for the microstrip with an anisotropic substrate, as studied in [8]. Following the same rationale as above, the electric field $E_z^{\mathrm{TE}_1}$ of the guided mode associated with the TE_1 surface wave of the background waveguide (the leakage field) is given by

$$E_z^{\mathrm{TE}_1} = -j\mathrm{Res}\left\{\tilde{G}_{zz}(k_{x,\mathrm{TE}_1}; k_z)\tilde{T}(k_{x,\mathrm{TE}_1})\right\} e^{-jk_{x,\mathrm{TE}_1}x}. \tag{10}$$

Making use of (23), given in the Appendix, the above residue is found to be proportional to the location of the surface-wave pole in the transverse wavenumber plane (rather than inversely proportional to it as before) so that

$$\mathrm{Res}\left\{\tilde{G}_{zz}(k_{x,\mathrm{TE}_1}; k_z)\right\} \propto k_{x,\mathrm{TE}_1} \tag{11}$$

as $k_z \to k_{\mathrm{TE}_1}$. Hence,

$$E_z^{\mathrm{TE}_1} \propto \tilde{T}(k_{x,\mathrm{TE}_1})k_{x,\mathrm{TE}_1}. \tag{12}$$

Thus, as $k_z \to k_{\mathrm{TE}_1}$ ($k_{x,\mathrm{TE}_1} \to 0$), it is found that $E_z^{\mathrm{TE}_1} \to 0$. Therefore, a transition of the type $C_0 \to C_1$ is possible even if $\tilde{T}(0) \neq 0$. This justifies that the dominant BM of a microstrip line can make a transition to an SFWLM solution provided that the leakage is in the form of the TE_1 surface wave of the background waveguide.

B. Transition From BM Solutions to Space-Leaky Solutions

A direct transition from a BM to a mode that leaks into both space and the TM_0 surface wave has never been observed previously. It has always been observed that the BM first becomes leaky by evolving into an SFWLM. An explanation of this general property can be given by considering the behavior of the solutions in the complex k_z-plane.

If a BM solution were to evolve directly into an SPWLM solution, the BM pole on the real axis would have to pass through the branch point at k_{TM_0}, and continue along the real axis to the branch point at k_0, before crossing the branch cut associated with the k_0 branch point. Such a hypothetical trajectory is shown as path (1) in Fig. 10. If this were possible, there would be a finite region of the real k_z-axis, $k_0 \le k_z \le k_{\mathrm{TM}_0}$, where the wavenumber is real and, therefore, there is no leakage into

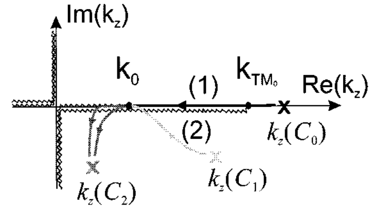


Fig. 10. Hypothetical trajectories showing: (1) a C_0 (BM) solution making a transition to a C_2 (space and surface-wave leaky) solution and (2) a C_1 (surface-wave leaky) solution making a transition to a C_2 solution.

the TM_0 surface wave. From (6), it appears that this is only possible if

$$\tilde{T}\left(\sqrt{k_{\mathrm{TM}_0}^2 - \beta^2}\right) = 0, \qquad \text{for } k_0 \le \beta \le k_{\mathrm{TM}_0} \tag{13}$$

which implies that

$$\tilde{T}(k_x) = 0, \qquad \text{for } 0 \le k_x \le \sqrt{k_{\mathrm{TM}_0}^2 - k_0^2}. \tag{14}$$

This means that there is a finite region of the k_z-axis for which the transform of the transverse profile function is identically zero. If it is assumed that the transverse profile function $T(x)$ has a transform that is an analytic function everywhere in the complex plane, then it must be identically zero. This follows from the property that a function that is analytic everywhere and zero on a finite length contour must be identically zero. Most common basis functions have this analytic property and, hence, the conclusion is expected to hold in most practical situations.

If the current of a BM is zero (such as for an odd dominant mode on coplanar strips or an odd higher order mode on a microstrip), the condition $\tilde{T}(0) = 0$ is satisfied and, therefore, (14) may be satisfied in an approximate sense provided that $k_{\mathrm{TM}_0} \approx k_0$. In this case, there may be an approximate continuity between the C_0 and the C_2 solutions. This is, for example, the case found for the transition of the EH_1 mode analyzed in [6, Fig. 4] and [9, Fig. 2]. Nevertheless, the curves presented for the microstrip analyzed in Fig. 5 show that the C_0 solution for the EH_1 mode continues to an improper real C_1 solution that meets another improper real C_1 solution, at which point a complex SFWLM solution then begins. This figure shows that a C_2 leaky solution appears to begin close to the point where the bound EH_1 mode evolves into one of the real improper C_1 modes. However, as mentioned previously, it has been carefully checked that there is not any *mathematical* connection between the C_0 and C_2 solutions, although this fact does not exclude a possible *physical* transition between the bound (C_0) and SPWLM (C_2) solutions.

C. Transition From a Surface-Wave Leaky Solution to a Space-Wave Leaky Solution

In general, it can also be stated that there cannot be mathematical continuity between an SFWLM and an SPWLM solution; i.e., a complex C_1 solution cannot turn into a complex C_2 solution. Since the above solutions are located on different sheets of the k_z Riemann surface, the only possible transition must be of the form shown as path (2) in Fig. 10. This transition requires that the C_1 solution emerge from the lower sheet of the TM_0 branch point, then passes through (or possibly around) the k_0 branch point, and then crosses the branch cut associated with the k_0 branch point. If the trajectory passes around the k_0 branch point instead of through it, there will be a frequency region for which the leaky mode will lie in the upper half of the k_z-plane, corresponding to a mode that grows in the direction of propagation, despite the fact that the mode would lie on the zero sheet of the k_0 branch point and the top sheet of the k_{TM_0} branch point and, hence, there would be no leakage. Such a mode would violate the conservation of energy. Hence, path (2) in Fig. 10 is the only allowed hypothetical path of transition. This implies that the solution has to be purely real (and bound) at $k_z = k_0$. However, a mode with this propagation wavenumber will leak into the TM_0 surface wave unless the condition

$$\tilde{T}\left(\sqrt{k_{\mathrm{TM}_0}^2 - k_0^2}\right) = 0 \tag{15}$$

is satisfied. This special condition is not expected to be found in general, and it certainly is not found when using the common basis functions $T(x)$ for the dominant mode on narrow strip problems. However, the condition may be approximately true for an odd mode such as the EH_1 mode for the case $k_{\mathrm{TM}_0} \approx k_0$. Hence, for an odd mode, there may be an approximate continuity between the C_1 and C_2 solutions, but not an exact continuity.

Despite the lack of an mathematical continuity found between the dominant C_1 and C_2 solutions, there could still be a *physical* continuity between these two leaky-wave solutions. Thus, it may happen that when a C_2 solution losses its *physical* significance by entering the region $\beta > k_0$, a C_1 solution with $k_0 < \beta < k_{\mathrm{TM}_0}$ may take over to continue the physical leakage of power. In this case, the nature of the leakage would change from leakage in the form of space and surface waves to leakage in the form of surface wave only. Results presented in Section IV will verify that this is indeed possible.

IV. Numerical Results

One of the most interesting points discussed in Section III was the fact that there is no mathematical continuity between the different leaky-mode solutions, although there may be a possible physical continuity between these modes. This point will now be numerically studied using the dispersion relations shown in Fig. 11 for a microstrip line with an isotropic substrate. Fig. 11(a) and (b) shows the normalized phase and attenuation constants of the C_0 bound dominant mode, a C_1 SFWLM solution, and a C_2 SPWLM solution. It is observed that the BM never becomes leaky (as expected from the discussion in Section III) and that the two leaky modes are two completely independent solutions that are never connected (also as expected from the discussion in Section III). According to the PCC discussed previously, the C_1 SFWLM solution can be physically significant in two frequencies ranges, where $k_0 < \beta < k_{\mathrm{TM}_0}$: between 13 and 14 GHz and above 18 GHz. The C_2 SPWLM solution can be physically significant where $\beta < k_0$, namely, between 14.8 and 18 GHz.

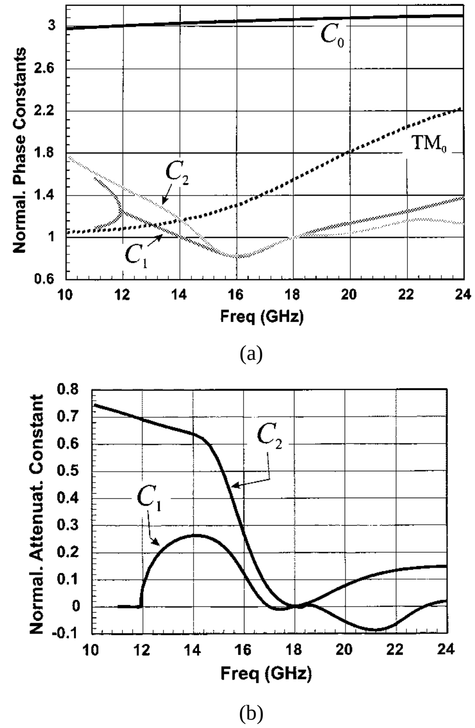
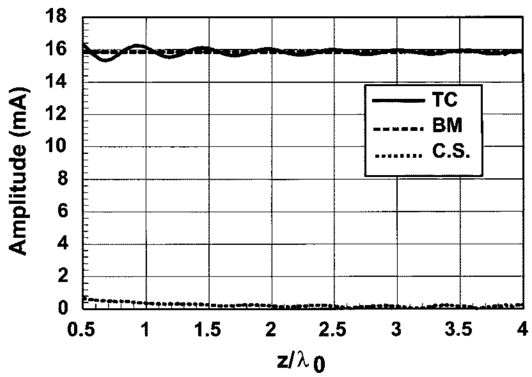


Fig. 11. Plots of the normalized phase constants for the C_0 (BM) solution, the C_1 (surface-wave leaky) solution, and the C_2 (space and surface-wave leaky) solution for a microstrip line on an isotropic substrate with $w = 4$ mm, $h = 1.27$ mm, and $\epsilon_r = 10.2$.

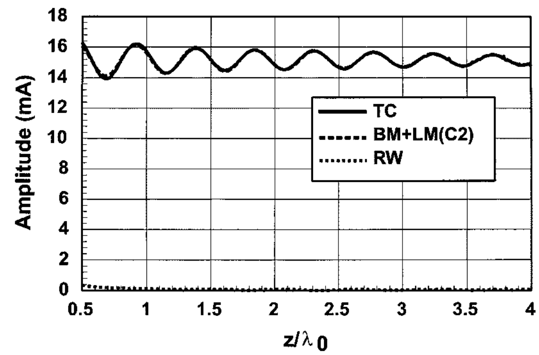
In order to study the physical significance of the different leaky-mode solutions and, hence, study any possible physical transition between them, the problem of current excitation from a 1-V delta-gap source will be used as the analysis tool, following the method in [27]. A study of the current excited from the source provides good physical insight into the nature of the excited modes and gives a clear indication of the qualitative and quantitative significance of the different modes. Specifically, the correlation between the current of a leaky mode, as defined by the corresponding residue in the complex k_z-plane (see Fig. 3), and the actual current on the line excited by the source is used to define the extent of the physical relevance of the leaky mode. To make the assessment of physical significance quantitative, a numerical modal decomposition of the line current $I(z)$ is obtained by application of the generalized pencil of matrices (GPOF) method [33] to $I(z)$. The degree of correlation between the amplitude and propagation wavenumber of a leaky mode (as defined by the corresponding residue) and the amplitude and wavenumber provided by the GPOF fit to the current data determines the degree of physical significance of the leaky mode. The GPOF values have been obtained using 400 sampling points over the plotted range of current in the figures, with the precision parameter set to -3 [33]. If the amplitude or wavenumber of some of the GPOF waves vary considerably as the GPOF parameters change, this indicates that these waves do not really model physical guided modes on the line. Whenever this situation occurs, the corresponding amplitude of the GPOF wave will be denoted by (-,-).

At 13.3 GHz, it can be seen in Fig. 11(a) that the C_1 solution is physical according to the PCC. To ascertain if this leaky-mode



Freq=13.3 GHz		k_z/k_0	Amplitude
Theory	C_0	3.0204	(-15.853,0.000)
	C_1	(1.0872,-0.2460)	(-3.8447,-0.4041)
	TM_0	1.1272	
GPOF		(3.0204,-0.0000)	(-15.869,-0.0000)
		(1.0805,-0.0655)	(-0.3516,-0.6982)

Fig. 12. Magnitude of the total current as well as the BM and the continuous-spectrum currents at 13.3 GHz for the structure analyzed in Fig. 11. The table shows the comparison between the GPOF and theoretical (residue) results for the modal decomposition at this frequency.



Freq=16.7 GHz		k_z/k_0	Amplitude
Theory	C_0	3.0565	(-15.162,0.000)
	C_2	(0.9037,-0.0570)	(-1.3114,-0.2281)
	TM_0	1.4121	
GPOF		(3.0567,-0.0001)	(-15.185,-0.0000)
		(0.9059,-0.0587)	(-1.3470,-0.2868)
		(1.3600,-0.1680)	(-,-)

Fig. 13. Magnitude of the total current as well as the current of the BM + leaky mode, and the "residual-wave" current at 16.7 GHz for the structure analyzed in Fig. 11. The table shows the comparison between the GPOF and theoretical (residue) results for the modal decomposition at this frequency.

solution is really present to a significant extent on the line, a plot of the different currents is shown in Fig. 12, and the GPOF results are presented in the table immediately below the plot. The total current on the line is plotted, along with the current of the BM and continuous spectrum (the leaky modes are part of the continuous-spectrum current [29]). At this frequency, it is seen that the continuous-spectrum current is very weak, meaning that no leaky modes are excited to any significant degree, despite the fact that the C_1 leaky mode is a physical mode.

The table of Fig. 12 show excellent agreement between the residue results (theory) and the GPOF results for both the amplitude and wavenumber of the BM, as expected, since the BM is always physical, and is also excited quite strongly here. For the C_1 leaky mode, there is reasonable agreement in the normalized phase constant (1.0872 versus 1.0805), but not in the attenuation constant (0.2460 versus 0.0655). Furthermore, the complex amplitude of the leaky mode predicted from the residue (−3.8447, −0.4041) does not agree at all with that from the corresponding GPOF wave (−0.3516, −0.6982). Hence, it is concluded that this leaky mode is not appreciably present on the line at this frequency, in spite of the fact that it is a physical mode in the sense of satisfying the PCC (the phase constant at this frequency is in the range $k_0 < \beta < k_{\mathrm{TM}_0}$). Part of the reason for this is the fact that the attenuation constant of the leaky mode is very high at 13.3 GHz, as seen in Fig. 11(b). Hence, this mode has a negligible value over most of the sampling region of the line, making it difficult for the GPOF method to accurately reconstruct this mode.

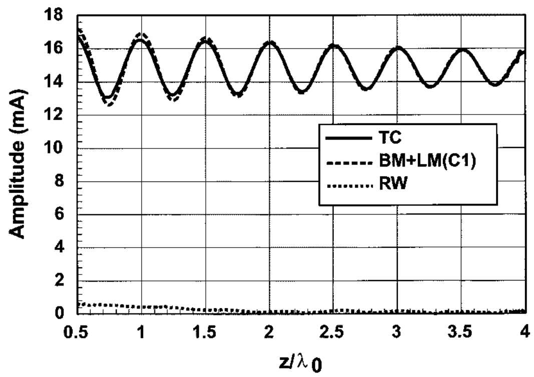
A similar situation (not shown here) was also found at 15 GHz. At this frequency, the leaky mode that is physical according to the PCC is the C_2 mode instead of the C_1 mode, which is clearly nonphysical at this frequency since it has a phase constant that is less than k_0. The large attenuation constant of the C_2 leaky mode [see Fig. 11(b)] causes this mode be significant only in a region very close to the source.

From the phase constants shown in the dispersion diagram of Fig. 11(a), it would be expected that the C_1 mode would be physically significant between 13–14 GHz, and the C_2 solution would be physically significant after about 14.4 GHz. However, because of the high attenuation constants at these frequencies, neither of the leaky modes are physically significant, except perhaps in a region very close to the source.

The situation changes at 17 GHz, where, according to Fig. 11(a) and (b), the C_2 leaky-mode solution satisfies the PCC and also has a small attenuation constant. This fact is reflected in Fig. 13, which shows that the sum of the BM and the C_2 leaky-mode current accounts almost entirely for the total current on the line. The results of the table included with this figure corroborate that the modal spectrum mainly consists of the BM and one C_2 leaky mode. The good correlation between the results of the GPOF fit and the theoretical (residue) values for the C_2 leaky mode points out the high degree of physical significance of this leaky mode. The absence of the C_1 leaky mode in the total current confirms the lack of physical significance of this mode at this frequency.

The plots and table of Fig. 14 show the situation at 19 GHz, where only the C_1 leaky mode is expected to be physical according to the PCC. Both the plots and results of the table of Fig. 14 show that the C_1 leaky mode is significant at this frequency.

Thus, although there is no mathematical continuity between the C_2 (SPWLM) solution and the C_1 (SFWLM) solution in the dispersion plot of Fig. 11, there exists a physical continuity for the leakage of power in the sense that, as the frequency increases from 16.7 to 19 GHz, the leaky mode that is physically present on the line changes from the C_2 to C_1 mode. This physical transition is expected based on the fact that the C_1 solution loses physical validity while the C_1 solution gains physical validity as the frequency increases over the range indicated. The nature of the physical continuity between different leaky-mode



Freq=19 GHz		k_z/k_0	Amplitude
Theory	C_0	3.0718	(-14.829,0.000)
	C_1	(1.0827,-0.0408)	(-2.6737,-0.24944)
	TM_0	1.4121	
GPOF		(3.0719,-0.0001)	(-14.859,-0.0000)
		(1.0819,-0.0335)	(-2.2914,-0.1802)
		(1.0612,-0.2142)	(-,-)
		(1.5871,-0.0845)	(-,-)

Fig. 14. Magnitude of the total current as well as the current of the BM + leaky mode, and the “residual-wave” current at 19 GHz for the structure analyzed in Fig. 11. The table shows the comparison between the GPOF and theoretical (residue) results for the modal decomposition at this frequency.

solutions will, in general, be dependent on the particular type of structure and the source excitation.

V. Conclusions

The evolution and allowable transitions of leaky modes on printed-circuit lines has been studied. A theory has been developed to understand the different possible mathematical transitions from a BM to a leaky mode, or from one type of leaky mode to another. Although most of the discussion and all of the results have been focused on a microstrip line, it is expected that the conclusion that were reached are general. These conclusions explain many of the modal transitions that have been observed on printed-circuit lines in the literature. A summary of the specific conclusions is listed below. References are also given to indicate where a particular type of modal transition has been observed.

1) The dominant (quasi-TEM) BM never becomes a leaky mode for a microstrip line on an isotropic substrate [24].
2) The dominant (quasi-TEM) BM may transition to a leaky mode on those structures where the current has odd symmetry (e.g., coplanar strips or coplanar waveguides) [7], [12], [14], [16].
3) Higher order modes on microstrip line may transition into leaky modes. All modes with odd current symmetry may make this transition [6], [9], [26]. Even modes may also make the transition, provided that the total current vanishes as the wavenumber approaches that of the TM_0 mode (Fig. 6).
4) For a microstrip line on an *anisotropic* substrate, the dominant BM can transition into a leaky mode, which then leaks in the form of the TE_1 surface wave of the background waveguide [8], [12].
5) When a BM transitions into a leaky mode, the leaky mode must be of the type where leakage occurs into only the surface wave of the background waveguide [12], [26], not of the type where leakage also occurs into space. A direct transition from a BM to a space-leaky mode is not possible.
6) A transition from an SFWLM to a mode that leaks into both space and the surface wave is not possible. The two types of leaky modes always remain separate solutions.

Although no mathematical continuation can exist between different types of leaky-mode solutions, as mentioned in 6), there may exist a physical continuation between the leaky modes. That is, one type of leaky mode may lose physical significance and disappear from the total current on the line that is excited by a practical source, while another type of leaky mode may become significant to take its place. Hence, there may be a physical continuation in the leakage of power as frequency changes, although the two types of modes remain separate solutions that are not mathematically connected. This behavior has been demonstrated by showing numerical results for the current on a microstrip line excited by a delta-gap source.

Appendix

For an isotropic substrate, the zz component of the SDGF can be written as [34]

$$\tilde{G}_{zz}(k_x, k_z) = -\frac{1}{k_t^2}\left(\frac{k_x^2}{D_{\mathrm{TE}}} + \frac{k_z^2}{D_{\mathrm{TM}}}\right) \tag{16}$$

where

$$D_{\mathrm{TE}} = \frac{1}{j\omega\mu_0}\left(u_0 + u\coth uh\right) \tag{17}$$

$$D_{\mathrm{TM}} = \frac{j\omega\varepsilon_0}{u_0 u}\left(u + \varepsilon_r u_0 \coth uh\right) \tag{18}$$

and $u_0 = \sqrt{k_t^2 - k_0^2}$, $u = \sqrt{k_t^2 - \varepsilon_r k_0^2}$ with $k_t^2 = k_x^2 + k_z^2$.

The values of k_t making D_{TE} and D_{TM} zero are precisely the wavenumbers of the TE and TM modes of the background waveguide (surface waves), respectively. It can be seen that when k_t tends to one of these wavenumbers, k_{sw}^i (where superscript i represents either TE or TM), D_i can be approximated by the pole behavior in the horizontal wavenumber (k_t) plane as

$$\lim_{k_t \to k_{\mathrm{sw}}^i} \frac{1}{D_i} \propto A_i \frac{-2k_{\mathrm{sw}}^i}{k_t^2 - {k_{\mathrm{sw}}^i}^2} \tag{19}$$

where, for convenience, the factor $(-2k_{\mathrm{sw}}^i)$ has been included in the numerator so that the residue at $k_t = k_{\mathrm{sw}}^i$ is simply the appropriate constant A_i. In the transverse wavenumber (k_x) plane, the approximation can be written as

$$\lim_{k_x \to 0} \frac{1}{D_i} \propto A_i \frac{-2k_{\mathrm{sw}}^i}{k_x^2 - {k_{x,\mathrm{sw}}^i}^2}. \tag{20}$$

The residue of the term $1/D_i$ in the k_x-plane can then easily be found to be

$$\mathrm{Res}_x^i = A_i \frac{k_{\mathrm{sw}}^i}{k_{x,\mathrm{sw}}^i}. \tag{21}$$

As $k_z \to k_{\mathrm{TM}_0}$ and, therefore, $k_{x,\mathrm{sw}}^i = k_{x,\mathrm{TM}_0} \to 0$, the residue in the k_x-plane approaches infinity. In particular, from (21) and (16), it is seen that the residue approaches

$$\mathrm{Res}_x^{\mathrm{TM}_0} \sim -A_{\mathrm{TM}_0} \frac{k_{\mathrm{TM}_0}}{k_{x,\mathrm{TM}_0}}. \tag{22}$$

In the TE case, as $k_z \to k_{\mathrm{TE}_1}$ and, therefore, $k_{x,\mathrm{sw}}^i = k_{x,\mathrm{TE}_1} \to 0$, the residue in the k_x-plane approaches zero because of the extra k_x^2 term in (16). In this case, the residue approaches

$$\mathrm{Res}_x^{\mathrm{TE}_1} \sim -A_{\mathrm{TE}_1} \frac{k_{x,\mathrm{TE}_1}}{k_{\mathrm{TE}_1}}. \tag{23}$$

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